

In[19]:= **f = Sqrt[x^2 + y^2] + Sqrt[1 - x^2] + Sqrt[1 - y^2]**

Out[19]=  $\sqrt{1 - x^2} + \sqrt{1 - y^2} + \sqrt{x^2 + y^2}$

In[20]:= **gradijent = D[f, {{x, y}}**

Out[20]=  $\left\{ -\frac{x}{\sqrt{1 - x^2}} + \frac{x}{\sqrt{x^2 + y^2}}, -\frac{y}{\sqrt{1 - y^2}} + \frac{y}{\sqrt{x^2 + y^2}} \right\}$

In[21]:= **Tacke = Solve[gradijent == 0, {x, y}]**

Out[21]=  $\left\{ \left\{ x \rightarrow -\frac{1}{\sqrt{2}}, y \rightarrow 0 \right\}, \left\{ x \rightarrow \frac{1}{\sqrt{2}}, y \rightarrow 0 \right\}, \right.$   
 $\left. \left\{ x \rightarrow 0, y \rightarrow -\frac{1}{\sqrt{2}} \right\}, \left\{ x \rightarrow 0, y \rightarrow \frac{1}{\sqrt{2}} \right\}, \left\{ x \rightarrow -\frac{1}{\sqrt{3}}, y \rightarrow -\frac{1}{\sqrt{3}} \right\}, \right.$   
 $\left. \left\{ x \rightarrow \frac{1}{\sqrt{3}}, y \rightarrow -\frac{1}{\sqrt{3}} \right\}, \left\{ x \rightarrow -\frac{1}{\sqrt{3}}, y \rightarrow \frac{1}{\sqrt{3}} \right\}, \left\{ x \rightarrow \frac{1}{\sqrt{3}}, y \rightarrow \frac{1}{\sqrt{3}} \right\} \right\}$

In[22]:= **Hodf = Simplify[D[f, {{x, y}, 2}]**

Out[22]=  $\left\{ \left\{ -\frac{1}{\sqrt{1 - x^2}} + \frac{1}{\sqrt{x^2 + y^2}} + x^2 \left( -\frac{1}{(1 - x^2)^{3/2}} - \frac{1}{(x^2 + y^2)^{3/2}} \right), -\frac{x y}{(x^2 + y^2)^{3/2}} \right\}, \right.$   
 $\left. \left\{ -\frac{x y}{(x^2 + y^2)^{3/2}}, -\frac{1}{\sqrt{1 - y^2}} + \frac{1}{\sqrt{x^2 + y^2}} + y^2 \left( -\frac{1}{(1 - y^2)^{3/2}} - \frac{1}{(x^2 + y^2)^{3/2}} \right) \right\} \right\}$

In[23]:= **HodfM = MatrixForm[Hodf]**

Out[23]//MatrixForm= 
$$\begin{pmatrix} -\frac{1}{\sqrt{1 - x^2}} + \frac{1}{\sqrt{x^2 + y^2}} + x^2 \left( -\frac{1}{(1 - x^2)^{3/2}} - \frac{1}{(x^2 + y^2)^{3/2}} \right) & -\frac{x y}{(x^2 + y^2)^{3/2}} \\ -\frac{x y}{(x^2 + y^2)^{3/2}} & -\frac{1}{\sqrt{1 - y^2}} + \frac{1}{\sqrt{x^2 + y^2}} + y^2 \left( -\frac{1}{(1 - y^2)^{3/2}} - \frac{1}{(x^2 + y^2)^{3/2}} \right) \end{pmatrix}$$

In[24]:= **HodfuTacke = Hodf /. Tacke**

Out[24]=  $\left\{ \left\{ \{-2 \sqrt{2}, 0\}, \{0, -1 + \sqrt{2}\} \right\}, \left\{ \{-2 \sqrt{2}, 0\}, \{0, -1 + \sqrt{2}\} \right\}, \right.$   
 $\left. \left\{ \{-1 + \sqrt{2}, 0\}, \{0, -2 \sqrt{2}\} \right\}, \left\{ \{-1 + \sqrt{2}, 0\}, \{0, -2 \sqrt{2}\} \right\}, \right.$   
 $\left. \left\{ \left\{ -\sqrt{\frac{3}{2}}, -\frac{\sqrt{\frac{3}{2}}}{2} \right\}, \left\{ -\frac{\sqrt{\frac{3}{2}}}{2}, -\sqrt{\frac{3}{2}} \right\} \right\}, \left\{ \left\{ -\sqrt{\frac{3}{2}}, \frac{\sqrt{\frac{3}{2}}}{2} \right\}, \left\{ \frac{\sqrt{\frac{3}{2}}}{2}, -\sqrt{\frac{3}{2}} \right\} \right\}, \right.$   
 $\left. \left\{ \left\{ -\sqrt{\frac{3}{2}}, \frac{\sqrt{\frac{3}{2}}}{2} \right\}, \left\{ \frac{\sqrt{\frac{3}{2}}}{2}, -\sqrt{\frac{3}{2}} \right\} \right\}, \left\{ \left\{ -\sqrt{\frac{3}{2}}, -\frac{\sqrt{\frac{3}{2}}}{2} \right\}, \left\{ -\frac{\sqrt{\frac{3}{2}}}{2}, -\sqrt{\frac{3}{2}} \right\} \right\} \right\}$

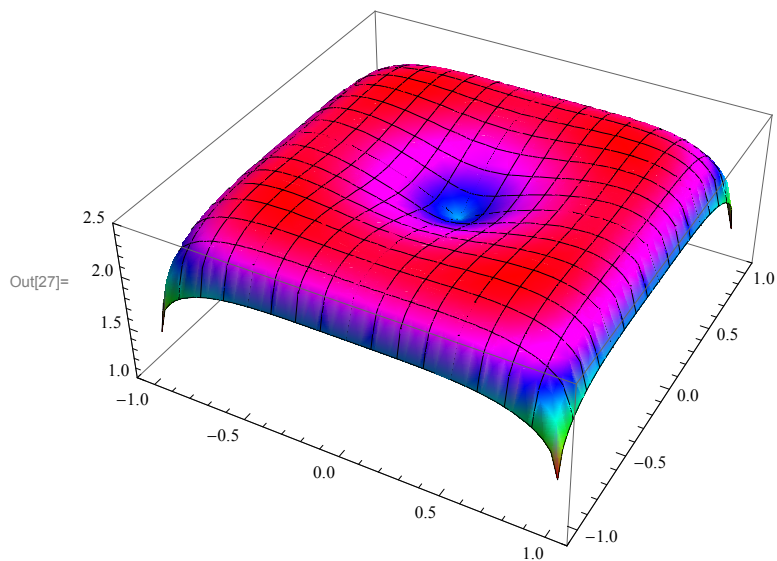
In[25]:= **minf = Map[PositiveDefiniteMatrixQ, HodfuTacke, 1]**

Out[25]= {False, False, False, False, False, False, False, False}

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In[26]= maxf = Map[PositiveDefiniteMatrixQ, -HodfuTacke, 1]
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Out[26]= {False, False, False, False, True, True, True, True}
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In[27]= Plot3D[f, {x, -1, 1}, {y, -1, 1}, ColorFunction -> Hue,  
PlotRange -> {{-1.1, 1.1}, {-1.1, 1.1}, {1, 2.5}}]
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In[28]= Show[%27, ImageSize -> Full]
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